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THE UNIVERSITY OF SASKATCHEWAN
Department of Mathematics

Mathematics 210.03 (Sections 01, 03, and 05)

Time: 3 hours Final Exam December 14, 1985

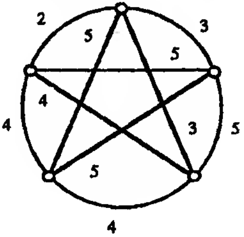
Instructions:

- Your name, section, and ID number should appear on all work handed in. Use an HB pencil or equivalent to indicate your choices for part one on the optical mark scan sheet.
- There are 3 marks for each multiple choice question. All other marks are indicated in the margin for a total of 88 marks.
- No aids are permitted. (For purposes of this exam, calculators are not considered to be an aid.)
- Attempt all questions.

Part I

Each question is worth 3 marks. Select one answer for each question.

1. The weight of any minimal spanning tree of the graph



is:

a) 10 b) 11 c) 12 d) 13 e) 14 f) 15

2. Indicate which of the following four logical arguments are correct.

(A)	(B)	(C)	(D)
$p \to (q \vee r)$	$\neg p \to \neg q$	$\neg p \to q$	$(p \wedge q) \to r$
$\neg q \wedge p$	q	p	p
$\therefore\ r$	$\therefore\ p$	$\therefore\ \neg q$	$\therefore\ q \to r$

a) A and D are valid. b) B and D are valid c) A and C are valid
d) A, C and D are valid. e) C and D are valid f) B, C, and D are valid.

3. If $p \to q$ is false, then the value of $\neg(p \wedge q) \to q$ is:

a) False b) True c) No conclusions

4. Consider the following relations R on a set A

(A) A is any set, and xRy iff x=y.
(B) $A = \wp(S)$, where S is any set, and xRy iff $y \supseteq x$.
(C) $A = \{1,2,3\}$: $R = \{(1,1), (2,2), (3,1), (1,3)\}$
(D) $A = Z^+$, the set of positive integers; xRy iff $x \mid y$.

One of the following propositions pertaining to to (A) … (F) is true. Which one?

a) (A), and (B) are partial orderings. b) (A), (B), and (D) are partial orderings.
c) (B), and (D) are partial orderings. d) (B), (C), and (D) are partial orderings.
e) None of the above.

5. The Karnaugh map for the Boolean function corresponding to the truth table:

x	y	z	f(x,y,z)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

a)

x \ y z	00	01	11	10
0	●			●
1		●		●

b)

x \ y z	00	01	11	10
0	●		●	●
1		●		

c)

x \ y z	00	01	11	10
0		●	●	
1	●			●

d)

x \ y z	00	01	11	10
0	●	●	●	
1	●			

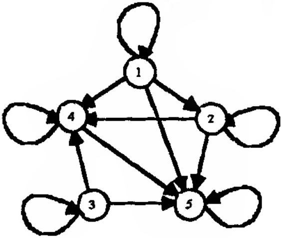
e)

x \ y z	00	01	11	10
0		●		●
1	●			●

f)

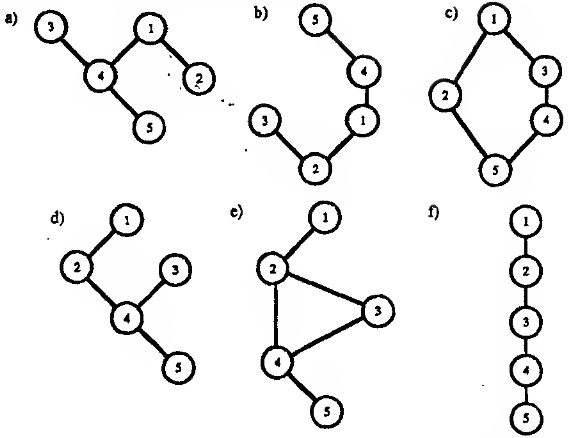
x \ y z	00	01	11	10
0	●		●	
1	●			●

6. Consider the digraph below representing a partial order on the set A = {1,2,3,4,5}.



(continued...)

The Hasse diagram corresponding to this poset is:



7. The translation of *"At least one train is faster than every car."* into logical notation is:

(U ≡ "everything", C(x) ≡ "x is a car", T(x) ≡ "x is a train", F(a,b) ≡ "a is faster than b")

a) $\exists y[T(y) \land \forall x[\, C(x) \to F(y,x)]]$
b) $\exists y[T(y) \to \forall x[\, C(x) \to F(y,x)]]$
c) $\exists x[C(x) \to \forall y[\, T(y) \land F(y,x)]]$
d) $\exists x[C(x) \land \forall y[\, T(y) \to F(y,x)]]$
e) $\forall y[T(y) \land \exists x[\, C(x) \to F(y,x)]]$
f) $\forall y[T(y) \to \forall x[\, C(x) \to \neg F(y,x)]]$

8. The proposition $\neg[\forall x(\, P(x) \land (x \le k)\,)]$ can be simplified to:

a) $\forall x(\, P(x) \lor (x<k)\,)$ b) $\exists x(\, \neg P(x) \lor (x>k)\,)$ c) $\exists x(\, P(x) \land (x>k)\,)$
d) $\forall x(\, P(x) \lor (x<k)\,)$ e) $\forall x(\, \neg P(x) \lor (x<k)\,)$ f) $\exists x(\, \neg P(x) \land (x \ge k)\,)$

9. One of the following propositions is false. Indicate which one.

   a) If S is finite and $f: S \to R$ is a one to one then $|S| \geq |R|$.

   b) The trial particular solution for the non-homogeneous recursion $x_{n+1} - 4x_n = 4^{n+1}$ should be $An4^n$.

   c) $\exists\delta\forall\varepsilon P(\varepsilon,\delta) \Leftrightarrow \forall\varepsilon\exists\delta P(\varepsilon,\delta)$

   d) In a Boolean algebra, $\forall a[a \wedge \neg a = 0]$.

10. The transitive closure R^+ of the relation { (3,4), (4,5), (2,3), (1,2), (1,4), (1,5), (2,5), (3,5) } is:

    a) { (1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5) }
    b) { (1,2), (1,4), (1,5), (2,3), (2,5), (3,4), (3,5), (4,5) }
    c) { (1,1), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5) }
    d) { (1,1), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5), (2,1), (3,2), (4,3), (5,4) }
    e) { (1,2), (1,3), (1,4), (1,5), (2,3), (2,5), (3,4), (3,5), (4,5) }
    f) { (1,2), (2,1), (1,4), (4,1), (1,5), (5,1), (2,3), (3,2), (2,5), (5,2), (3,4), (4,3), (3,5), (5,3), (4,5), (5,4) }

11. The homogeneous solution for the recurrence equation $f_n = 3f_{n-2} + 2f_{n-3} + 12(2^n)$ is of the form:

    a) $(An+B)n^2$ b) $An\,2^n$ c) $(An^2 + Bn + C)(-1)^n + D(2)^n$
    d) $2n^2$ e) $(An+B)(-2)^n + C$ f) $(An + B)(1)^n + C(2)^n$
    g) $(An + B)(-1)^n + C(2)^n$ h) $4n + C$

12. Which of the following is a linear difference equation of order 6?

    a) $y(n+1) - y(n-5) = 5n^3$ b) $y(n+2) - 3y(n+1) = 5n^3$
    c) $t(n/3) + 4t(n) = 3^n$ d) $y(n+1) - y(n) = 2y(n-1) - 5y(n-2)$
    e) $t(n+3) + 4t(n) = 2t(n-2)$ f) $r_{n+3} = 3r_{n-1} + n^2 + 1$

13. Let S = { (a,1,A), (b,2,B), (c,2,D) }, be a relation defined on the Cartesian product $X \times Y \times Z$, and let T = { (α,E), (β,F), (χ,D)} be a relation defined on the Cartesian product $U \times Z$. Which of the following corresponds to *the join of S and* T ?

    a) { (a,1,A), (b,2,B), (c,2,D), (α,E), (β,F), (χ,D) }.

    b) { (c,2,D), (χ,D)}

    c) { (c,2,D,χ)}

    d) { (a,1,A,α,E), (b,2,B,α,E), (e,2,D,α,E), (a,1,A,β,F), (b,2,B,β,F), (c,2,D,β,F), (a,1,A,χ,D), (b,2,B,χ,D), (e,2,D,χ,D)}

    e) { (e,2,D,χ),(a,1,D,χ),(b,2,D,χ)}

14. Which of the following forms of mathematical induction is most appropriate for proving that the solution of a 3rd order linear recurrence is correct?

a) $[\, P(k) \wedge \forall n((P(n) \to P(n+1)) \vee (n<k))] \longrightarrow \forall n(P(n) \vee (n<k))$

b) $[\, P(k) \wedge \forall n((P(k) \wedge P(k+1) \wedge \ldots \wedge P(n) \to P(n+1)) \vee (n<k))] \longrightarrow \forall n(P(n) \vee (n<k))$

c) $[\, P(k) \wedge P(k+1) \wedge P(k+2) \wedge \forall n((P(n) \wedge P(n+1) \wedge P(n+2) \to P(n+3)) \vee (n<k))] \longrightarrow$
$\forall n(P(n) \vee (n<k))$

d) None of the above.

15. For the recurrence equation $f(n+3) - 4f(n+2) - f(n+1) + 4f(n) = 4n + 3$, the guess for the trial particular solution should be:

a) $(An+B)n^2$	b) $An\,4^n$	c) $An^2 + Bn + C$	d) $4\,n^2$
e) $(An+B)(-2)^n$	f) $An\,2^n$	g) $An + B$	h) $4\,n + 3$

16. Which of the following statements is not true for the Boolean algebra $(K, \cdot, +, \neg)$?

a) Every a in K has a unique complement.
b) for all a and b in K, $ab = ba$ and $a+b = b+a$
c) for all a and b and c in K, $a(b+c) = ab + ac$, and $a+(bc) = (a+b)(a+c)$
d) for all a and b in K, $a(\neg a + b) = \neg a + b$.
e) for all a and b and c in K, if (ab) and (bc) are in K then so is (ac).
f) for all a and b in K, $a(\neg a + b) = ab$
g) for all a in K, $a(\neg a) = 0$, and $a + \neg a = 1$.
h) for all a and b in K, $\neg(a+b) = \neg a \neg b$

17. Which Boolean expression realizes the truth table given below?

C	A	R	f(A,C,R)
1	0	0	0
0	1	0	1
1	1	0	0
1	1	1	1
0	0	1	1

a) $(\neg A + \neg C + R)(A + \neg C + R)$
b) $(A + \neg C + \neg R)(A + C + R)(\neg A + \neg C + \neg R)$
c) $\neg A \neg C R + A \neg C R$
d) $\neg A \neg C \neg R + AC\neg R + \neg ACR$
e) R

18. The binary relation R = { (x,a), (y,a), (y,b), (z,b), (a,w), (b,w) } describes a partial order on the set {a,b,w,x,y,z}. A total order of these same elements can be described by listing the elements left to right in a list. Which of the following total orders is not a linear extension of R?

a) [x,y,z,a,b,w] b) [y,x,z,a,b,w]
c) [z,x,y,b,a,w] d) [x,b,y,a,z,w]
e) [z,y,x,a,b,w] f) [z,x,y,a,b,w]

Part II

19. Complete any two of the following proofs. [10 marks]

a) Let G be a graph with minimum degree (n-3)/4. Prove that G has at most 3 components.

b) Write xRy for the relation " *there is a path between* x *and* y ". Show that R is an equivalence relation. Describe the equivalence classes.

c) A rooted ternary tree is a tree in which the root has degree at most 3, and all other vertices have degree at most 4. Prove that, for such trees, the number of leaves is at most 3^h where h is the height of the tree. (Interpret the tree consisting of just a root vertex to be of height 0.)

d) Prove, by induction, that the solution to the difference equation $y_n = -3y_{n-1} - 2y_{n-2}$ with $y_0 = 2,\ y_1 = 2,$ is $6(-1)^n - 4(-2)^n$.

20. a) Find a complete solution to the difference equation

$$y_n = 7y_{n-1} - 6y_{n-2}.$$

given the initial values $y_0 = 0$, $y_1 = 1$.

[5 marks]

b) Find the complete solution to the difference equation

$$y_n = 3y_{n-1} + 10y_{n-2} + 5^n$$

for $y_0 = 0$, and $y_1 = 0$, given that the homogeneous solution is of the form $A(5)^n + B(-2)^n$.

[5 marks]

21. a) Determine the disjunctive normal form for f where f: $B^3 \dashrightarrow B$, is a Boolean function defined by

$$f(x, y, z) = \neg[\neg(x \vee y) \vee (\neg(x) \wedge z)].$$

[5 marks]

(21 continued...)

b) Minimize the Boolean function $f(w, x, y, z) = w'xy'z' + w'xy'z + wxy'z' + wxy'z + w'x'yz' + w'xy\,z'$ by using the Karnaugh map technique. (a' denotes the complement of a)

[5 marks]

c) How many Boolean functions are there with 4 arguments? (ie. how many functions $f: B^4 \rightarrow B$ are there, where B is the set $\{0,1\}$?)

[2 marks]

d) If $p_1, p_2, \ldots, p_n$ are distinct literals and the well formed formula p contains at least one occurrence of each literal p_i, $1 \leq i \leq n$, how many rows are needed to construct the truth table for p?

[2 marks]